JMEPEG (2005) 14:123-131
DOI: 10.1361/10599490522248

1059-9495/$19.00

Design of Forging Process Variables under Uncertainties

Jalaja Repalle and Ramana V. Grandhi

(Submitted July 18, 2004)

Forging is a complex nonlinear process that is vulnerable to various manufacturing anomalies, such as variations in billet geometry, billet/die temperatures, material properties, and workpiece and forging equipment positional errors. A combination of these uncertainties could induce heavy manufacturing losses through premature die failure, final part geometric distortion, and reduced productivity. Identifying, quantifying, and controlling the uncertainties will reduce variability risk in a manufacturing environment, which will minimize the overall production cost. In this article, various uncertainties that affect the forging process are identified, and their cumulative effect on the forging tool life is evaluated. Because the forging process simulation is time-consuming, a response surface model is used to reduce computation time by establishing a relationship between the process performance and the critical process variables. A robust design methodology is developed by incorporating reliability-based optimization techniques to obtain sound forging components. A case study of an automotive-component forging-process design is presented to demonstrate the applicability of the method.

Keywords forging process design, random process variables, reliability assessment, reliability-based optimization, uncertainty quantification

1. Introduction

This research begins by exploring the possible sources of uncertainties in the hot-forging process. A brief description of the various sources of uncertain parameters in the forging process is provided. A robust design methodology is developed that considers the randomness in the process parameters. Uncertainty Quantification (UQ) and Reliability-Based Optimization (RBO) are the two tools that are used in a robust design method. The effectiveness of the proposed methodologies is demonstrated with applications to automotive and aerospace components. Furthermore, this research opens a new era for incorporating uncertainty analysis in the conventional forging process design to improve product quality and reliability, and to reduce the total manufacturing cost.

Many complex industrial and military components, as well as many consumer goods, are produced through forging processes. Forging is a plastic deformation process in which a simple cylindrical shape, either hot or cold, is transformed through a number of stages to a predetermined shape, primarily by compressive forces exerted by dies (Ref 1). In recent years, the forging industry has become an increasingly competitive global marketplace. As such, customers in this industry have placed considerable pressure on the manufacturers to decrease development and production costs. Fewer physical prototypes and shorter development times lead to a less costly design process. Increased tool/die life decreases the number of work stops in the production, thereby reducing the overall production costs; these costs can quickly become significant over large production lots or when working with expensive materials. Tool or die life in the forging operation is dependent on three criteria: the mechanical properties of the die material, process variables, and the operating conditions. One way to improve tool life is to reduce the load on the tool by modifying the forging process, tool design, preform design, or forging stages.

Yoshinari (Ref 2) discussed the causes for tooling damage and the steps that are required to improve die life. According to Yoshinari, damage to the die occurs through surface adhesion, and plastic deformation, wear, or breakage. Lubrication type, material properties, die/material interface conditions, forging loads, thermal loads, and process conditions can help to predict the die damage state. Better lubrication and process conditions, and the reduction of forging loads will improve die life. In a conventional design, all of the forging process parameters are considered to be deterministic and constant. However, there is always some uncertainty involved in forging process variables. These variables include material properties, process, and environmental conditions. A combination of these uncertainties could induce heavy manufacturing losses through premature die failure, final-part geometric distortion, and production risk. Identifying the sources of uncertainties, and then quantifying and controlling them, reduces risk in the manufacturing environment and minimizes the overall cost of production.

Generally, uncertain parameter information can be obtained either as sparse data points, intervals, expert opinions, or as probability distributions. Uncertainties in the forging processes come from both quantitative and qualitative sources. Quantitative or noncognitive sources are related to randomness in physical observations, which come from process condition variations, such as randomness in friction, stroke length, and billet temperature. Qualitative or cognitive sources are related to the skill or experience of the operator and the conditions of the machinery. Depending on the nature of the sources, the uncertainties in forging can be classified into four categories, as shown in Fig. 1. They are:

Jalaja Repalle and **Ramana V. Grandhi,** 209 RC, Department of Mechanical and Materials Engineering, 3640 Colonel Glenn Highway, Wright State University, Dayton, OH 45435. Contact e-mail: rgrandhi@cs.wright.edu.

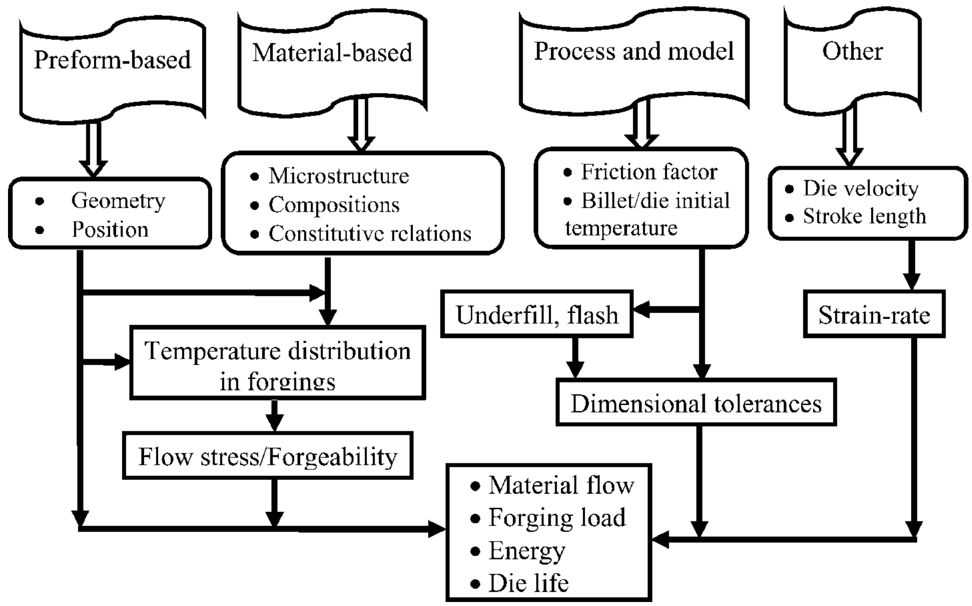

Fig. 1 Sources of uncertainties in forging process

- Preform-based uncertainties
- Material-based uncertainties
- Process and model parameter uncertainties
- Other miscellaneous uncertainties

1.1 Preform-Based Uncertainties

These uncertainties include variations related to the preform such as variations in preform geometry, heating times in the furnace, temperature profiles, transfer times from the furnace to the die, and alignments.

1.2 Material-Based Uncertainties

These uncertainties consist of variations in the workpiece and die material compositions, microstructure, elastic, plastic, and thermal properties.

1.3 Process and Model Parameter Uncertainties

These uncertainties include variations in process and model parameters, such as friction coefficients at the interface, heat transfer rates, and stroke length, due to the repetitive use of lubrication, inconsistent cooling rates, and machinery fatigue conditions. These uncertainties also arise from variations in the lubrication system (i.e., spray angle, time, pattern, and the speed).

1.4 Other Miscellaneous Uncertainties

Some of the other miscellaneous uncertainties include errors in tooling assembly, human intervention, and environmental conditions.

Reliability and durability depend on the process performance in the presence of these uncertainties. In conventional design, the effect of these uncertainties is considered by empirical safety factors. These safety factors were derived based on experience but do not guarantee safety or optimum performance. Hence, these designs are susceptible to risks, such as premature die failure, incomplete die fill, and reduced product quality. Therefore, in this research, an attempt is made to quantify these parameter uncertainties. UQ and RBO are the tools that are used to quantify the uncertainties in producing designs that meet safety requirements. The optimization techniques that are developed in this research help to design robust processes more economically and more quickly than traditional approaches.

2. Robust Design Methodology

A process that is insensitive to noise in variables is described as being robust. For instance, a manufacturing process may exist to make a product (e.g., metal wheels), but, despite best efforts, the product quality varies widely. This variation occurs due to process parameters such as friction (due to the repeated use of a lubricant), initial billet temperature (due to heat transfer that occurs in transferring the billet to dies from the furnace), die velocity, and stroke length (due to machine backlash errors.) Rather than tightening up tolerances on the process parameters, it may be advantageous to adjust the level of design parameters to reduce random parameter sensitivity (i.e., robust design under uncertainties). This way a product or process is achieved that is not only of a high quality, but gives consistently high quality.

Robust design methodology (Fig. 2) of the forging process consists of the following steps: screening critical variables, evaluating the sources of uncertainties and their probability distributions, UQ through variability, reliability-assessment methods, and RBO.

The forging process reliability and outcome depend on a number of parameters such as initial billet temperature, friction

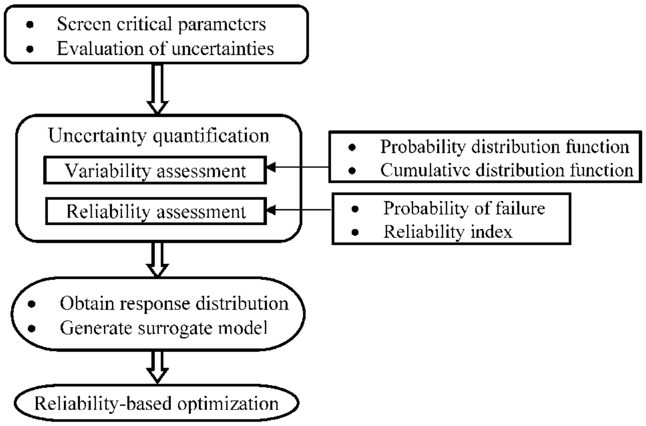

Fig. 2 Robust design methodology

factor, die temperature, stroke length, and die velocity. Among these parameters, the critical ones have to be identified, which is done using design-of-experiments (DOE) techniques. A response surface model (RSM) is generated and used to represent the process behavior. Variation in the process response is estimated by applying Monte Carlo simulations (MCSs) on the RSM. Cumulative distribution functions (CDFs) are generated to represent system variability. Process reliability is evaluated by estimating the probability of failure and the reliability index (RI) of the behavior. To make the process robust, parameter uncertainties are considered in the design by applying reliability constraints through RBO.

3. Example Case Study

An axisymmetric metal wheel having an H-cross section (Fig. 3a) is considered for the case study. The disk is axisymmetric with rib height H and rib width B, as shown in the finite-element model in Fig. 3(b). The complexity of the forging process increases with the H/B ratio. In this example, the ratio is taken as 1. A horizontal symmetry is assumed; hence, a quarter model and only the top die are considered for further analysis.

Today, sophisticated tools are available to simulate the total forging process. A finite-element analysis package, DEFORM 2D (Ref 3), is used to simulate the hot-forging process and to predict the forging loads, metal flow, and deformation patterns. In most hot-forging processes, the requirements are to improve the tool life and the product quality by reducing the forging load and by designing an economical and robust process. In this study, the forging process using a mechanical press is considered. For this process, the ram speed is much higher compared with the ram speed of the hydraulic press; therefore, the heat generation due to the deformation is high. It increases the temperature at the center of the billet by 100 to 150 °C. However, for the die/billet interface, this effect is offset by the die-chilling effect. Therefore, for the contact boundary surface, the dominant factor is the heat conduction between the billet and the die. Because the die temperature is much lower than that of the billet, the heat loss due to the conduction between these two bodies is very large, and there exists a severe temperature gradient. The boundary temperature of the billet is reduced by about 200 °C due to the die-chilling effect. This causes a large temperature variation between the center of the billet and the die contact boundary of the billet. Therefore, in this study, temperature effects are considered by conducting a nonisothermal forging simulation of an axisymmetric H-cross-section wheel. Because the forging process is nonisothermal, there exists a temperature variance on the die surface. Forging dies are usually heated to temperatures as high as 250 to 400 °C to reduce die chilling. The effective strain-rate is directly dependent on the nodal velocities. Therefore, it is an instantaneous variable and can be directly influenced by the die velocity. Hence, die velocity is taken as one of the variables.

In a hot-forging process, the billet is heated in the temperature range of 925 to 1300 °C in the furnace and then transferred to the forging dies. In the process of transference, the heat-transfer flow occurs from the billet to the atmosphere. As a result, the effective temperature of the billet varies from the furnace temperature. This variation is not constant for all of the parts. Furthermore, it has a strong effect on the microstructure/service properties of the product. Therefore, the billet temperature is considered to be one of the random parameters.

One of the other important random variables is the friction factor. Variations in the friction factor arise due to the repetitive use of lubrication and from lubrication system conditions. Additionally, variations occur in the stroke length and die velocity due to the existence of any backlash errors in the tooling assembly, or due to errors in the operating equipment. These variations determine the final dimensional accuracies and rejection rates. Ambient temperature variations can cause variations in the heat-transfer rates of forging billets and dies, thereby affecting the final product quality. Hence, these parameters are considered as random parameters for further forging process optimization. Among all of these uncertainties, the forging load is sensitive to only some of the variables. These are the critical random variables to the forging load, and they have to be identified through screening methods.

3.1. Screening the Variables

Factorial screening methods are used to identify the contribution of each random parameter to the response of the system (Ref 4). Because this research focuses on the forging load, six input variables that affect the forging load are identified. They are: initial billet and die temperatures, die velocity, friction factor, stroke length, and environmental temperature. The sensitivity of each parameter for the response is analyzed through analysis of variance (ANOVA) (Ref 5) and DOE analysis. Using a two-level fractional factorial DOE, the simulation points are generated. The forging load for each DOE design point is computed, and the main effect of each process parameter is evaluated. Together, the analyses yield a Pareto plot (Fig. 4), which enables the identification of the variables that significantly affect the forging load.

From the Pareto plot, it is shown that the billet temperature and the friction factor together contribute 70% of the overall response to the forging load. Thus, these two variables are considered in the evaluation of the variability and reliability of the process. Die velocity and stroke length are the other sig-

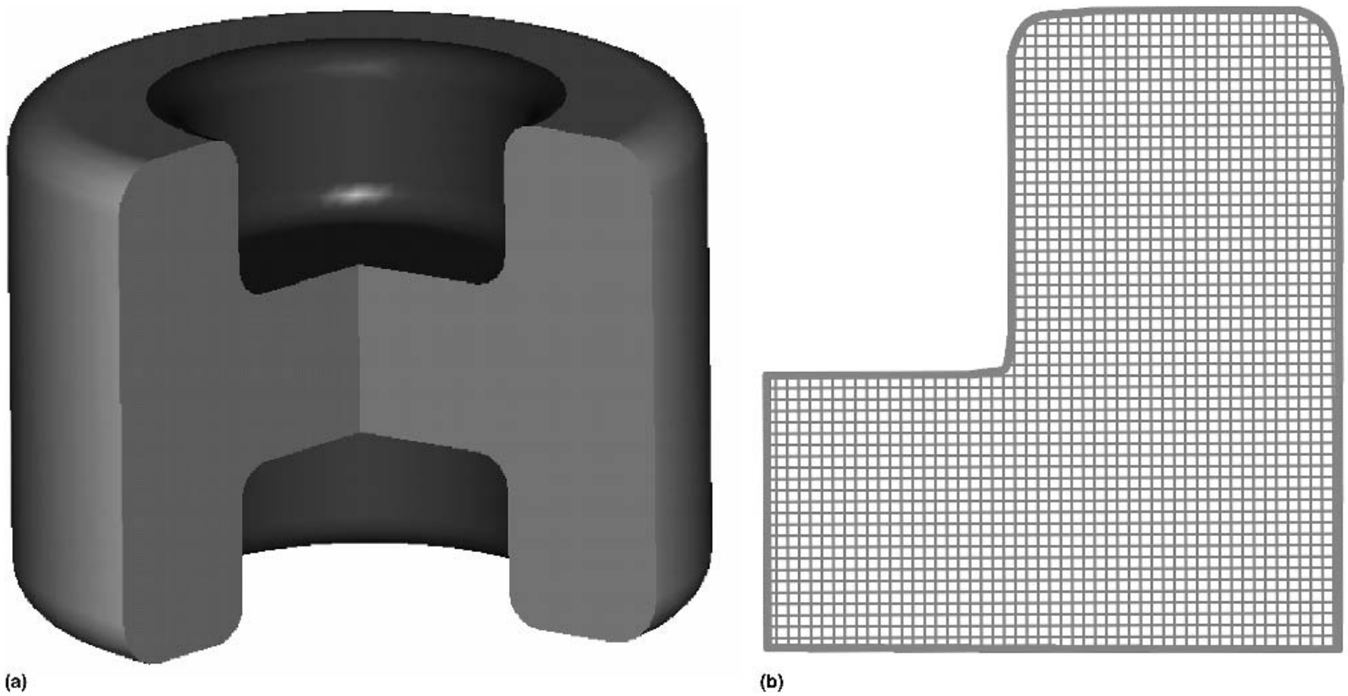

Fig. 3 (a) Axisymmetric wheel. (b) One-quarter finite-element model

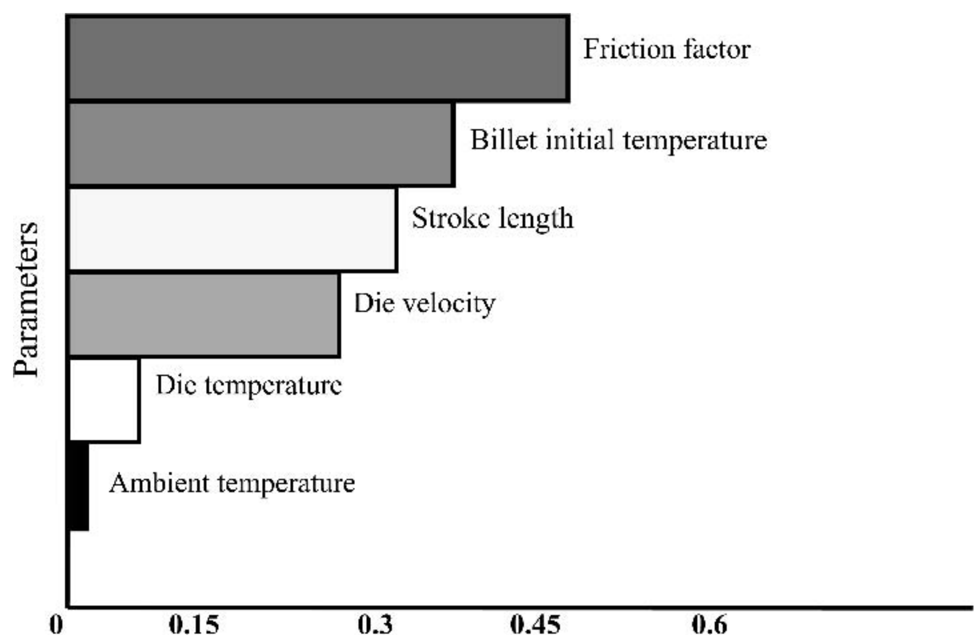

Fig. 4 Pareto plot for forging load

nificant random parameters, which are also considered in the forging process design. RSM is generated in terms of these two critical variables, while fixing the remaining variables at some likely values.

3.2. Response Surface Model

Once the critical parameters are screened, an RSM is generated to represent the process response (Ref 6). RSM is a polynomial used to represent an empirical relationship between the response of the system and the critical design parameters. This is used when there is no explicit relationship between design variables and responses or when such relationships are complicated. It filters the numerical noise present in the analysis. The polynomial coefficients are estimated by using the method of least squares, which consists of minimizing the sum of squares of the differences between responses and their expected values. The responses are computed at design points.

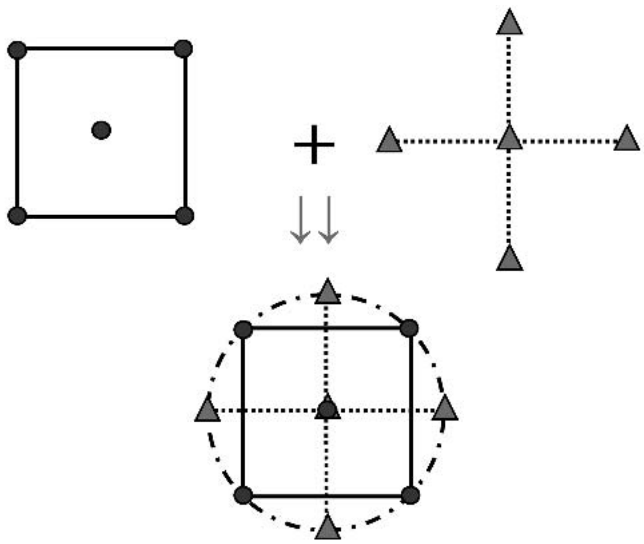

Fig. 5 Central composite design

These design points are selected using the DOE factorial methods. Among all of the factorial methods, the central composite design (CCD) method offers a satisfactory alternative to a full factorial design.

A CCD (Fig. 5) contains an embedded factorial or fractional factorial design with center points that are augmented with a group of "star points," which allow estimation of the curvature behavior of the system performance (Ref 3), and the quadratic terms are efficiently estimated through the axial points. Hence, this method is used for selecting the design points in this study. The total number of simulations required for the CCD method is $2^k + 2k + N$, where k is the number of parameters and N is the number of center points. The quadratic polynomial is constructed for the system performance, and mathematically it is written as:

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j(i<j)}^{k} \hat{\beta}_{ij} x_i x_j \quad \text{(Eq 1)}$$

where β_i are the regression coefficients for the linear terms, β_{ii} are the coefficients for pure quadratic terms, β_{ij} are the coefficients for the cross-product terms, x_i, and x_j are random parameters, and $x_i x_j$ denotes interactions between two random parameters.

The development of the surrogate model avoids repetitive finite-element simulations for studying the uncertain behavior. Using this RSM, the approximate response variability is calculated for variations in random parameters.

3.3. Variability Assessment

The uncertainties associated with the process are usually accounted for in the form of variability distributions for random-process variables. In principle, the distributions for these random variables could be, for example, uniform, normal, lognormal, or Weibull. However, the selection of a particular type of probability distribution depends on a number of factors:

- Available format of the data
- The nature of the problem
- The underlying assumptions associated with the distribution
- Convenience and simplicity for further computations

The normal distribution is the most commonly used distribution for the random variables. Hence, the normal distribution is assigned for the random variables in this research. A brief background of the normal distribution is provided here.

3.3.1 Normal Distribution. The normal distribution (also known as the Gaussian distribution) is a symmetric and bell-shaped density curve for a random parameter. The normal distribution is characterized by two parameters: the mean, μ; and the standard deviation, σ. The mean is a measure of center, and the standard deviation is a measure of spread. The normal probability density function (PDF) of the normal distribution of a random parameter is shown in Fig. 6, where the random variable x can take any value from $-\infty$ to $+\infty$.

A probability distribution is defined for each one of the random variables. The cumulative effect of all these distributions causes the overall probability distribution of the response function. After the RSM is developed for the process performance, or objective, the effect of uncertain variables can be incorporated into the model through the use of MCS. MCS is, effectively, a random number generator that creates values for each uncertain parameter. Values are chosen within the limits for each variable and with a frequency that is proportional to the shape of the probability distribution associated with each variable. The uncertain variables, billet temperature and friction factor, are assigned with a probability distribution within their limits used in the RSM. Variability assessment is done for each of these distributions. In a practical manufacturing process, the uncertain variables could have a correlation between them (e.g., billet temperature variations affect the film thickness of lubrication, which affects the friction factor at the interface). The correlation among these uncertain variables can be defined with the help of a correlation coefficient (CC). A CC is a number between 0 and 1. If there is no relationship between the variables, the CC is 0 or very low. As the strength of the relationship increases, the CC also increases. If they have a strong relationship, that is, if they are dependent on each other, the CC is 1. In the forging operation, the relationships among the variables are uncertain. Hence, the process variability is assessed for normally distributed random variables with different levels of CCs, starting from CC = 0.

3.3.2 Response Variability. In this case, randomness of the input random variables is assumed as a normal distribution with a zero mean and 10% variance. Normally distributed random numbers are generated using MCS. Using RSM, the forging load variability is computed for different levels of correlations of the normally distributed parameters. The response variation follows a normal distribution with different means and variances for different correlation levels of random variables. A CDF is obtained by integrating the area under the PDF. A CDF is defined as the probability that the variable

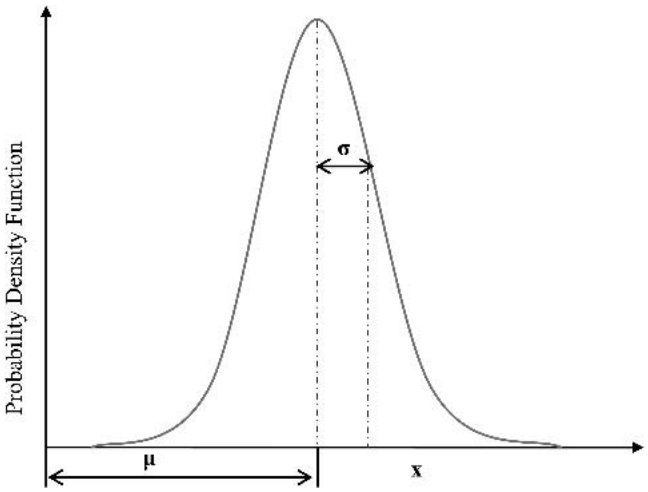

Fig. 6 Normal distribution

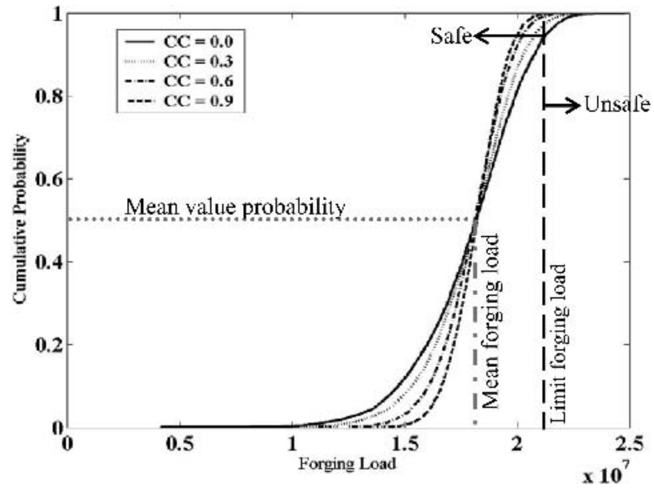

Fig. 7 CDF for forging load at different correlation coefficients (CCs)

takes a value less than or equal to x_{limit}. For a continuous function, this can be expressed as:

$$F = \int_{-\infty}^{x_{\text{limit}}} f(x)dx \qquad \text{(Eq 2)}$$

where $f(x)$ is the PDF.

The response CDFs for different correlation levels are plotted in Fig. 7. The y-axis represents the probability, and the x-axis is the allowable domain for the given probability function. The forging load corresponding to a cumulative probability of 0.5 represents the mean value of the forging load. The mean forging load is almost the same for all of the CC values, but the shape varies (Fig. 7). This means that the distribution varies with the correlation level. The CDF curve is skewed toward the right as the correlation among random variables decreases. The solid line indicates a completely independent case, which occupies a wider range of the response and has a large variance in the response. For all other correlation levels, this range, which shows a steeper distribution with less variance, is small. The straight dotted line in Fig. 7 represents the limit load in forging. The region to the right of the limit load is the failure region, and the region to the left is the safe region. Thus, process reliability is obtained from the cumulative probability of the response in safe region (Fig. 8). The reliability estimation method from the response variation and limit load is explained briefly in the following section.

3.4. Reliability Assessment

Process reliability is the probability that a system can perform its intended function for a specified interval under specified conditions. Generally, reliability analysis uses the limit-state function to evaluate the probability of process failure by determining whether the limit-state functions are greater or less than zero. One of the common limit-state functions in forging is the load function (Fig. 8), which can be written as:

$$g(x) = \text{limit load} - \text{actual load} \qquad \text{(Eq 3)}$$

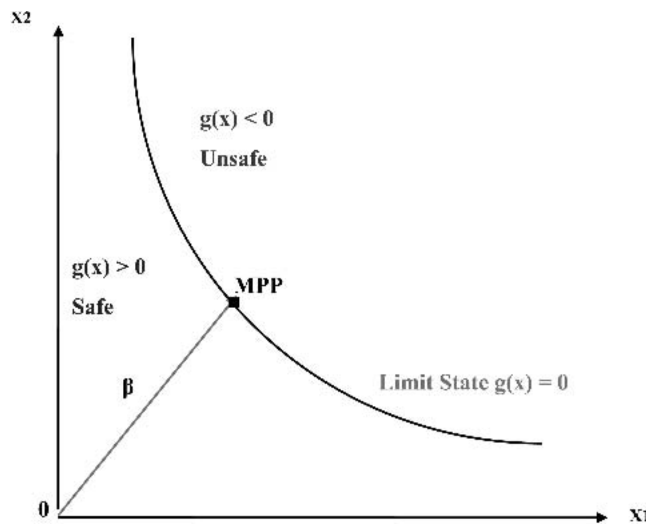

Fig. 8 Reliability index

Here the forging load is an RSM, a function of random parameters. The limit-state function $g(x) = 0$ is the boundary between safe and unsafe processes (Ref 7). The failure of the process occurs if the actual load exceeds the press limit load. Hence, probability of failure (P_f) is defined as:

$$P_f = P[g(x) < 0] \qquad \text{(Eq 4)}$$

It is computed as the integration of the joint PDF over the failure region. P_f can also be mentioned with the use of a standard normal CDF as follows:

$$P_f = \Phi(\beta) \qquad \text{(Eq 5)}$$

where Φ is the standard normal cumulative density function and β is the RI. β is the minimum distance from the origin to the design point on the limit-state surface, as shown in Fig. 8. It defines the number of standard deviations from the mean point accounted for in the failure probability prediction. This minimum distance point is called the most probable failure

Table 1 **P_f and RI for different correlation levels of normal random parameters**

Distribution	Correlation coefficient (CC)	Probability of failure (P_f)	Reliability index (RI)
Normal	0.0	0.1310	1.125
	0.3	0.0849	1.379
	0.6	0.0415	1.736
	0.9	0.0191	2.073

point (MPP). The MPP represents the worst combination of stochastic variables with the highest probability of failure.

It is apparent from the above definition of reliability that if the failure line, or limit-state line, is closer to the origin, the failure region is larger, and if it is farther away from the origin, the failure region is smaller. Thus, the position of the limit-state surface relative to the origin is a measure of the reliability of the process under assumed parameter distributions. In practice, the size of the random variables can be very large, and their distributions may vary as uniform, normal, Weibull, or extreme value distributions. The limit-state surface changes its position and shape due to changes in input random variables distribution and their correlation. Therefore, there exist different failure regions for each of the distributions. Hence, the probability of failures (P_f) and the RIs are computed for normal distribution with different levels of correlation factors, as shown in Table 1, to explore the design space.

It is observed from the above results (Table 1) that P_f decreases as the correlation between the variables increases for normally distributed random variables. That means that as the correlation increases, the limit surface is moved away from the origin, thereby shifting the forging load away from the limit load. Hence, the reliability of the process, or the forging tool life, increases. However, these results are not unique. In practice, the correlation levels among the variables and their uncertainty distribution are unknown and are greatly problem-dependent. Therefore, these results provide a guideline for a reliability analysis and an optimization of the forging process. From this study, it is apparent that the independent random variables are the worst design situation to consider in the forging optimization. Hence, in further optimization, all the random variables are assumed to be independent in nature. In general, the number of random parameters is not limited to two. Therefore, in the following sections, the forging process optimization is carried out with all four critical random variables: initial billet temperature, friction factor, forging die velocity, and stroke length.

4. Design Optimization

Design optimization is a tool that is applied in the forging process to obtain, for example, the minimum forging load, power consumption, strain variance, or scrap (Ref 8). There has been considerable work done in forging optimization (Ref 9-11) in which the process simulations were based on the deterministic information of various variables. A deterministic-based optimization (DBO) solution does not consider variability, and, as a result, the design solution may be risky or overly conservative. Therefore, there is a need to use an RBO to increase the process robustness. An RSM is used in the optimization of the forging process under uncertainty. To date, there has been no literature found on RBO application for the metal-forging process. However, RBO techniques have been widely used in structural optimization (Ref 12). In this research, the reliability analysis is extended to nonlinear metal-forging processes. The developed methodology is applied to an axisymmetric metal wheel-forging optimization. Additionally, the relative advantages of the RBO over the DBO are presented.

4.1. Deterministic-Based Optimization

It is important to control the material flow behavior during the forging to ensure the uniformity of deformation and to complete die filling. Furthermore, the control of strains and strain variance in the deformed product helps in the development of “favorable” mechanical and microstructural properties. This goal can be achieved by optimizing the initial billet temperature, friction factor, forging die velocity, and stroke length. Constraints are placed on the underfill to ensure complete die-fill with no defects and on the forging load. The uniformity of deformation is always a critical factor in the quality of the final product and in the distribution of properties through the material being deformed. The effective strain variance is thus chosen as the objective function. The response surface approach is used to approximate the objective function and constraints. In the most generic sense, the deterministic optimization problem can be stated mathematically as:

Objective: minimize effective strain variance $\bar{\varepsilon}_{var}$:$f(x_i)$

Subject to: $g_1(x_i) \leq 0$ i.e., underfill ≤ 0

$g_2(x_i) \leq 0$ i.e., forging load $\leq$ allowable forging load

Design variables: $x_i = 1,2,3,4$

where x_1 is the initial billet temperature, x_2 is the friction factor, x_3 is the die velocity, and x_4 is the stroke length. The normalized side bounds on design variables are $-1 \leq x_i \leq 1$, which are determined based on the process requirements and the forge press capacities. Here, $\bar{\varepsilon}_{var}$ is the weighted effective strain variance in the billet given by:

$$\bar{\varepsilon}_{var} = \frac{\sum_{i=1}^{n} A_i(\bar{\varepsilon}_i - \bar{\varepsilon}_{avg})^2}{\sum_{i=1}^{n} A_i} \quad \text{(Eq 6)}$$

where A_i is elemental area, n is the total number of elements, and $\bar{\varepsilon}_{avg}$ is the average strain in the workpiece, which is given by:

$$\bar{\varepsilon}_{avg} = \frac{\sum_{i=1}^{n} A_i\bar{\varepsilon}_i}{\sum_{i=1}^{n} A_i} \quad \text{(Eq 7)}$$

Table 2 DBO and RBO design solutions

Design parameters, x_i	Initial design	DBO	RBO
Billet temperature, °C	1150.0	1260.2	1078.5
Friction factor	0.55	0.37 (≅0.4)	0.35 (≅0.3)
Die velocity, mm/s	212.3	154.5	163.2
Stroke length, mm	67.0	66.8	66.3

Table 3 DBO and RBO results

Criterion	DBO	RBO
Strain variance	0.162	0.252
Maximum strain	4.721	6.732
Strain-rate variance	977.5	3604.6
Maximum strain-rate	443.8	4174.6
Load, tons	2065.2	1639.4
Probability of failure	55.0×10^{-4}	0.8×10^{-4}

Here, x_i design variables are considered as deterministic values, and $f(x)$ and $g(x)$ are the quadratic polynomial approximations in terms of the design variables x_i. The objective function and constraints surrogate models are used in the design optimization toolbox. By optimizing the forging variables, in addition to minimizing the effective strain variance, complete die fill and forging load is maintained at less than the limit load. For non-linear problems such as forging, the solution is dependent to some extent upon the "initial guess" vector. In this work, different varieties of guess vectors are tried, and the optimum value is given in Table 2.

4.2. Reliability-Based Optimization

In traditional deterministic optimization, the inherent variability present in the design variables and other variables is not considered. Due to inherent variability, the realized optimum design might be susceptible to a high probability of failure (Ref 13). In a reliability analysis, a constraint equation is written as a limit-state function $g(x)$, where $g(x)$ = forging load − allowable load. Due to the uncertainties in the design variables, $g(x)$ is a random variable itself. As a result of uncertainties, it is not certain whether $g(x)$ falls into the safe region or the failure region for an arbitrary value of variables. Therefore, the reliability measure P_f is used as a constraint. In this optimization problem, the forging load is taken as the random performance. Therefore, the P_f for the forging load is written as follows (Eq 4):

$$P_f = P[g(x) < 0]$$

A surrogate RSM model is used to fit the structural response in terms of random variables for RBO. An MCS is used to determine the failure probability of the process on the RSM. In practice, it is difficult to construct individual probability functions due to the scarcity of statistical data. Hence, an approximate normal distribution with 10% variance is assumed for the random variables, and a probability analysis is used. To obtain the optimum mean values of random variables, an optimization problem is formulated. In addition to a reliability constraint on the forging load, a deterministic constraint is placed on the underfill. The effective strain variance is the objective function the same as in the DBO. Unlike in DBO, objective and constraints functions are RSMs in terms of parameter distribution means. Mathematically, the reliability optimization problem can be written as:

Objective: Minimize effective strain variance $\bar{\varepsilon}_{var}$: $f(\mu_{xi})$

Subject to:

Deterministic constraint: $g_1(\mu_{xi}) \leq 0$ i.e., underfill ≤ 0

Reliability constraint: $P_f \leq P_{limit}$ (P_f due to forging load $\leq$ 0.0001)

Design Variables: μ_{xi} i = 1,2,3,4

where μ_{x1} is the mean value of initial billet temperature, μ_{x2} is the mean value of friction factor, μ_{x3} is the mean value of die velocity, and μ_{x4} is the mean value of stroke length. The normalized side bounds on mean values of design variables are $-1 \leq \mu_{xi} \leq 1$, which are determined based on the process requirements and the capacity of the forge presses. Here, $\bar{\varepsilon}_{var}$ is the weighted strain variance, and $g_1(x)$ is the deterministic constraint on the underfill in the billet, approximated in terms of the distribution mean values of the variables. The P_f of the forging process due to forging load is computed by using MCS on RSM.

In RBO, the optimizer checks the reliability constraints for every design parameter mean value and finds a feasible direction. RBO provides an optimized design solution for a desired reliable criterion. DBO and RBO design solutions are shown in Table 2.

It can be seen from the results that the optimum points are quite different in DBO and RBO. The optimum initial billet temperature (1260.2 °C) and friction factor (0.4) in DBO are higher than the RBO optimum temperature (1078.5 °C) and friction factor (0.3). The RBO optimum die velocity is higher than the DBO optimum die velocity. The RBO optimum stroke length is 0.5 mm less than the DBO stroke length, and this ensures a complete die fill, along with a reduction in the forging load even in the presence of variations. Various process performance variables are computed for both optimum designs and are tabulated in Table 3.

The RBO optimum design simulation gives higher effective strain and strain-rate variances, which implies that the RBO optimum design provides a lesser uniform material flow than the DBO solution. RBO also gives higher maximum strain and maximum strain rate values. However, the load for the RBO optimum design is reduced by 20% compared with the DBO. The reduction in forging load on the dies reduces the thermal and mechanical stresses in forging dies, and, therefore, improves the die life. The reduction in the energy required for the forging process facilitates the product to manufacture at lower press capacity, which in turn leads to a reduction in machinery investment cost.

However, these optimum designs may be subject to variations. Hence, a normal distribution with 10% variance is assumed at the mean optimum design, and the process reliability is then computed. By performing a probability analysis at the obtained DBO and RBO optimum points, a CDF plot for forg-

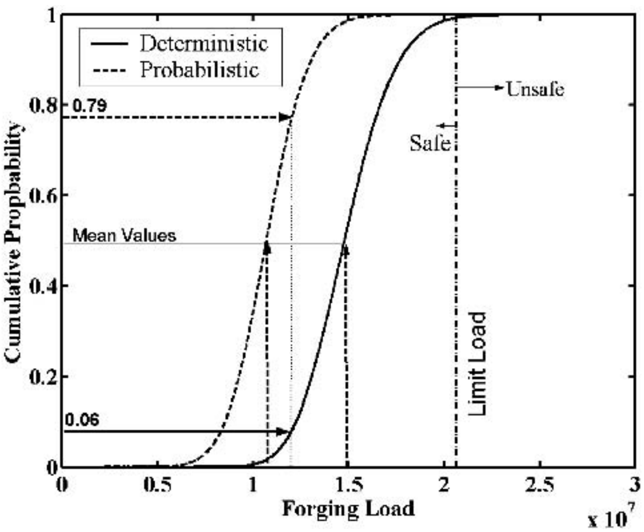

Fig. 9 CDF at DBO and RBO optimum solutions

ing load is generated. This plot is shown in Fig. 9. This CDF plot can be used to directly read the probability of failure of the forging process.

By performing an MCS on the limit state (i.e., the forging load) at DBO and RBO designs, the probability of failure is equal to 0.0055 and 0.00008, respectively (Table 3). The DBO case demonstrates that for every 10,000 parts produced, 55 of them will exceed the limit load. For the RBO case, only about 1 part in 10,000 will exceed the forging load.

From the plot (Fig. 9), it can be seen that the robust design solution of an RBO always yields a lower level of risk for the same randomness in the design variables. As a result, the optimum solution corresponds to a shift in the forging load distribution, which is to the left for both the PDF and the CDF.

From Fig. 9, the reduction in the performance variation can be seen for the RBO solution (dotted-line distribution). The most likely, or mean value, of the forging load is reduced from 1.5×10^7 to 1.1×10^7 N. At the forging load 1.2×10^7 N, the cumulative probability in the deterministic case is 0.06, while it is 0.79 in the probabilistic case. This shows that 6% of the deterministic outcomes are less than, or equal to, the forging load (1.2×10^7 N). The remaining 94% of outcomes exceed this load (i.e., 1.2×10^7 N), whereas in the probabilistic case 21% of the outcomes exceed this value. This illustrates that the RBO solution moves the response away from the limit load line. Therefore, the risk due to exceeding the forging load is lowered, thereby increasing the process reliability and improving the die life.

5. Summary Remarks

This research focuses on developing nontraditional design concepts based on probabilistic analysis and UQ techniques. The important aspects are the identification of critical random variables and their distribution. Various random variables in the forging process are evaluated using ANOVA and DOE screening methods. Critical random variables are assigned to normal distributions. A robust design methodology is developed by incorporating the randomness of random variables. This allows the quantification of uncertainties and leads to estimating the variability of the process.

This methodology is implemented on a generic axisymmetric H-cross-section metal wheel and can be adopted for any other forging operations. The optimization problem is solved to minimize the effective strain variance, while the forging load is used as a reliability constraint. The reliability-based optimum solution provides more strain variation and reduces the forging load. However, the cost savings due to the reductions in the forging load and energy are significant compared with the increased cost due to material flow. Moreover, reduced load in the RBO solution significantly reduces the probability of failure for the process. Therefore, this RBO reduces the manufacturing risk and improves the product quality. It also gives a more robust optimum point than the deterministic solution based on the selected reliability criterion. Additionally, if the forging process is designed by considering the uncertainties, then fluctuations in tooling loads can be greatly reduced, thereby improving the tool life.

Acknowledgment

This material is based upon work supported, in part, by the U.S. Department of Commerce, National Institute of Standards and Technology, Advanced Technology Program, and Cooperative Agreement Number 70NANB0H3014 (the Smartsmith project). Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the sponsor.

References

1. S. Kobayashi, S.I. Oh, and T. Altan, *Metal Forming and the Finite-Element Method,* Oxford University Press, 1989
2. T. Yoshinari, Trend in Process Tribology Focusing on Die Life Key Technology for Precise and Efficient Production, *R&D Review of Toyota CRDL,* Vol 34 (No. 4), 1999, p 1-10
3. Scientific Forming Technologies Corporation, *DEFORM 2D (Design Environment for FORMing) User Manual,* Multiple Operation Laboratory, Scientific Forming Technologies Corporation, Columbus, OH, 1999
4. T. Linda and M.C. Linda, "Finding Important Independent Variables through Screening Designs: A Comparison of Methods," presented at Proc. 2000 Winter Simulation Conference (Orlando, FL), 2000, p 749-757
5. E.P. Box and J.S. Hunter, *Statistics for Experiments,* John Wiley & Sons, 1978
6. A.J. Cornell, *How to Apply Response Surface Methodology,* ASQC, Milwaukee, WI, 1990
7. A. Haldar and S. Mahadevan, *Reliability Assessment Using Stochastic Finite Element Analysis,* John Wiley & Sons, 2000
8. J.S. Arora, *Introduction to Optimum Design,* McGraw-Hill, 1989
9. H. Cheng, R.V. Grandhi, and J.C. Malas, Design of Optimal Process Variables for Non-Isothermal Forging, *Int. J. Numer. Methods Eng.,* Vol 37 (No. 1), 1994, p 155-177
10. R.V. Grandhi, H. Cheng, and S.S. Kumar, Design of Forging Process Variables with Deformation and Temperature Requirements, *ASME J. Man. Sci. Eng.,* Vol 118 (No. 3), 1996, p 441-444
11. A. Kumar, R.V. Grandhi, A. Chaudhary, and D. Irwin, Modeling and Design of Control Variables in Metal Forming Processes, *Trans. CSME,* Vol 17 (No. 4A), 1993, p 613-631
12. S.V.L. Chandu and R.V. Grandhi, General Purpose Procedure for Reliability Based Structural Optimization under Parametric Uncertainties, *J. Adv. Eng. Soft.,* Vol 23 (No. 1), 1995, p 7-14
13. K. Irfan, C. McMahon, and M. Xianyi, "Reliability-Based Structural Optimization Using the Response Surface Method and Monte Carlo Simulation," presented at Eighth International Machine Design and Production Conference (UMTIK, Middle-East Technical University, Ankara, Turkey), Sept 1998